Dynamical mean-field theory of complex systems on sparse directed networks

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Since its inception about fifty years ago, dynamical mean-field theory (DMFT) has established itself as the foremost analytic method for studying the dynamics of many interacting elements (neurons, species, oscillators, etc) randomly coupled through nonlinear differential equations. Recently, there have been an impressive number of exciting applications of DMFT, particularly in the context of ecosystems and neural networks. Despite significant theoretical progress, DMFT is mainly limited to fully-connected networks, where each variable interacts with all others through Gaussian interactions. Nevertheless, the interactions in real-world complex systems are often *sparse* and *heterogeneous*, yet incorporating these features in the formalism of DMFT remains a formidable challenge. In this talk, I will outline the key steps to solve this problem and obtain an exact equation for the path-probability characterizing the effective dynamics of several models of complex systems on sparse directed networks. This new equation allows us to explore how network structure impacts the nonequilibrium dynamics of ecosystems, epidemic spreading, synchronization, and neural networks, thereby opening new avenues of research in several fields. As an example, I will discuss how to determine the phase diagram of the seminal neural network model of Sompolinsky *et al* [1] in the sparse regime, by combining numerical solutions of the path-probability equation with results from random matrix theory.

References

[1] H. Sompolinsky, A. Crisanti, and H. J. Sommers, "Chaos in random neural networks", Phys. Rev. Lett. **61**, 259-262 (1988).

[2] F. L. Metz, "Dynamical mean-field theory of complex systems on sparse directed networks", arXiv:2406.06346.

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