

Evaluation of the Lyapunov Exponent at Misiurewicz Points on the Logistic Map

FRANZ, Kleisson Spiess¹; FIDELIS, Antônio João¹.

¹ Instituto Federal Catarinense, IFC *campus* Rio do Sul.

The Logistic Map $x_{n+1} = rx_n(1 - x_n)$, $x_n \in [0, 1]$ and $r \in [0, 4]$, is one of the simplest and mostly widely known dynamical system. It shows a variety of behaviour present in more sophisticated systems [1]. From its critical point, one can generate a set of iterative functions called Supertracks Functions $s_n(r)$ [2]. These functions are bounds states of the dynamics of the bifurcation diagram of the logistic map. The intersection of these functions give birth to bifurcation points - for stable periodic orbits - and Misiurewicz Points - for unstable periodic orbits. Being unstable periodic orbits, it is expected that these points act as repeller of the orbits, but the observed behaviour is completely different: after some steps (called pre-period n), they pull the orbits to an apparently periodic p orbit that bursts in a chaotic regime and, after another steps, return to the apparently periodic orbit - these points are called $\mathcal{M}_{n,p}^d$ points, and d is the degenerescence in the orbit. We determine and compare the numerical and algebraic Lyapunov exponent of these Misiurewicz Points, and show the influence of the precision in the behaviour of the iteration process: the more precision is increased, the more detained the orbit becomes in the apparently periodic orbit.

References

- [1] May, R. Simple mathematical models with very complicated dynamics. *Nature* 261, 459–467 (1976). <https://doi.org/10.1038/261459a0>.
- [2] Oblow, E.M. Supertracks, supertrack functions and chaos in the quadratic map, *Physics Letters A*, Volume 128, Issue 8, 1988, Pages 406-412, ISSN 0375-9601, [https://doi.org/10.1016/0375-9601\(88\)90119-3](https://doi.org/10.1016/0375-9601(88)90119-3).
- [3] Romera, M.; Pastor, G.; Montoya, F. Misiurewicz points in one-dimensional quadratic maps, *Physica A: Statistical Mechanics and its Applications*, Volume 232, Issues 1–2, 1996, Pages 517-535, ISSN 0378-4371, [https://doi.org/10.1016/0378-4371\(96\)00127-6](https://doi.org/10.1016/0378-4371(96)00127-6).

Type

ORAL